Errata and remarks on *An Introduction to Ergodic Theory*

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This is a list of errors and remarks on *An Introduction to Ergodic Theory* (1st edition, 2000) by Peter Walters.

1 Errors

Page 33, line 4: $m(T^{-n_0}E\Delta T^{-n_0}A)$
Page 44, line 9: and such that $n \geq N_\epsilon$
Page 45, line 9: iff there is a subset
Page 50, Theorem 1.27: G must be assumed nontrivial.
Page 59, line 6: $T_i(X \setminus N) \subset X \setminus N$
Page 65, line 12: two-sided
Page 67, Theorem 2.13 (2): The proof uses Theorem 1.26, but Theorem 1.26 requires T to be invertible, and yet T is not assumed invertible in this proof.
This proof needs to assume T invertible.
The theorem works without assuming T invertible, but it needs a different proof. Refer to *Ergodic Theory: with a view towards Number Theory* by Manfred Einsiedler, Thomas Ward, Theorem 2.36.
Page 70, Lemma 3.3: Discreteness of H is useless. Delete that condition.

2 Remarks

Page 30, Theorem 1.10: A does not need to be assumed surjective. This is relevant later, for Page 50, Theorem 1.28.

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Page 46, Theorem 1.23, (3) $\Rightarrow$ (2)**:** This proof can be made a lot more elementary. $\forall f, g \in L^2(m)$, decompose $f = f_r + if_i, g = g_r + ig_i$, with $f_r, \cdots, g_i \in L^2_{\mathbb{R}}(m)$, then

$$(U_T^n f, g) = (U_T^n f_r, g_r) + (U_T^n f_i, g_i) + i(U_T^n f_i, g_r) - i(U_T^n f_r, g_i)$$

$$(f, 1), (1, g) = (f_r, 1)(1, g_r) + (f_i, 1)(1, g_i) + i(f_i, 1)(1, g_r) - i(f_r, 1)(1, g_i)$$

so if we can prove (2) for $L^2_{\mathbb{R}}(m)$, then it's done.

In real inner product space V, $\forall v, w \in V$, as one can check directly by expanding the terms:

$$(v, w) = \frac{1}{2}((v, v) + (w, w) - (v - w, v - w))$$

$$(v, 1)(1, w) = \frac{1}{2}((v, 1)(1, v) + (w, 1)(1, w) - (v - w, 1)(1, v - w))$$

So by decomposing the limit to three smaller limits, we get (2) from (3).

Page 50, Theorem 1.28: In the proof, it says that, if $\gamma, \delta \in \hat{G}$, and one of them is $\not\equiv 0$, then $(U_A^n \gamma, \delta) = 0$ eventually. But to prove this in the case where $\gamma, \delta \not\equiv 0$, we would have to show that $\lim_{n \to \infty} (U_A^n \gamma, \delta) = 0$.

Assume not, then since $U_A^n \gamma \in \hat{G}$, we must have $U_A^n \gamma = \delta$ for infinitely many n, thus $\exists k \geq 1, U_A^k \delta = \delta$.

Now, to proceed, we have to assume that T is surjective, then by Theorem 1.10, $\delta \equiv 1$, contradiction.

Page 51, Theorem 1.29: What is $\hat{B}$? It's undefined and I can't figure it out.